

# Health Investment over the Life-Cycle<sup>\*</sup>

Timothy J. Halliday<sup>†</sup>  
Department of Economics  
University of Hawai'i at Manoa  
& Institute for the Study of Labor (IZA)

Hui He<sup>‡</sup>  
Department of Economics  
University of Hawai'i at Manoa

and

Hao Zhang<sup>§</sup>  
Department of Economics  
University of Hawai'i at Manoa

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## Abstract

We study the evolution of health investment over the life-cycle by calibrating a model of endogenous health accumulation. The model is able to produce the decline in labor supply with age as well as the hump-shaped consumption profile. In both cases, health and health investment play a crucial role as the former encroaches upon healthy time and the latter crowds out non-medical expenditures as people age. Finally, we quantify the value of health as both an investment and a consumption good. We show that the investment motive is about three times higher than the consumption motive during the early 20s, but decreases over the life-cycle until it disappears at retirement. In contrast, the consumption motive increases with age and surpasses the investment motive during the mid 40s.

**Keywords:** Health Investment, Consumption motive, investment motive, life-cycle

**JEL codes:** E21, I12

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<sup>†</sup> Assistant Professor, Department of Economics, University of Hawai'i at Mānoa and IZA

<sup>‡</sup> Assistant Professor in the Department of Economics at the University of Hawai'i at Mānoa. Corresponding Author: E-mail: huihe@hawaii.edu

<sup>§</sup> PhD Candidate in the Department of Economics at the University of Hawai'i at Mānoa.

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Timothy J. Halliday<sup>†</sup>, Hui He<sup>‡</sup> and Hao Zhang<sup>§</sup>

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<sup>†</sup>Assistant Professor, Department of Economics, University of Hawai'i at Mānoa and IZA

<sup>‡</sup>Assistant Professor in the Department of Economics at the University of Hawai'i at Mānoa.  
Corresponding Author: E-mail: huihe@hawaii.edu

<sup>§</sup>PhD Candidate in the Department of Economics at the University of Hawai'i at Mānoa.

former encroaches upon healthy time and the latter crowds out non-medical expenditures as people age. Finally, we quantify the value of health as both an investment and a consumption good. We show that the investment motive is about three times higher than the consumption motive during the early 20s, but decreases over the life-cycle until it disappears at retirement. In contrast, the consumption motive increases with age and surpasses the investment motive during the mid 40s.

## 1 Introduction

To date, the literature on life-cycle economic behavior has largely been concerned with savings and consumption motives, but it has paid relatively less attention to the life-cycle motives for health-related behaviors and, particularly, expenditures on medical care. Indeed, there is a vast literature that has attempted to better understand whether and when consumers behave as buffer stock or certainty equivalent agents (*e.g.* Carroll (1997) and Gorinchas and Parker (2002)) as well as the extent to which savings decisions are driven by precautionary motives (*e.g.* Gorinchas and Parker (2002), Palumbo (1999), Hubbard, Skinner and Zeldes (1994)). Much of the earlier literature on these topics has been elegantly discussed in Deaton (1992). However, very little is known about the motives for expenditures on medical care

within a life-cycle context. This is true despite the fact that medical expenditures accounted for 13.9% of GDP in the US, 10.7% in Germany, 9.7% in Canada, and 7.6% in the United Kingdom in 2001 (see Exhibit 1 of Reinhardt, Hussey and Anderson (2004)). Moreover, in the US, it is estimated that 25% of medical expenditures by Medicare occur in the last year of life, so that there is a steep increase in these expenditures over the life-course (Hogan, Lunney, Gabel, and Lynn 2001). In this paper, we attempt to fill this void by investigating the life-cycle motives for expenditures on medical care.

We concern ourselves with two tasks. The first is to calibrate a life-cycle model of economic behavior with endogenous health accumulation and to use the calibrated model to better understand how labor supply, consumption, health investment and health interact over the life-course. We attempt to better understand how changes in health status affect other aspects of economic behavior using a structural framework. The second is to better understand how the motives for health investment change over the life-course.

Two motives for health investment were discussed in Grossman (1972a). The first is that individuals derive utility from being healthy. The second is that good health enables individuals to supply more labor either to the labor market or at home. The former reason is referred to as the “consumption motive” and the latter as the

“investment motive.” The relative importance of each of these motives will change over an individual’s life and, in particular, as people age, health will gradually move from an investment good to a consumption good. Indeed, for the young and healthy, the marginal utility of good health is low and the number of years they still have to live is high, and so for them, the consumption motive is low and the investment motive is high. In contrast, for the old and frail, the opposite is true; their health investment is primarily driven by the consumption motive. While this discussion is a direct qualitative implication of the Grossman Model, little if anything is understood about how the motives for and returns to health investment evolve over the life-course in the *quantitative* sense. This is one of the first papers to shed light on this issue.

As a precursor to what is to follow, we summarize our results. First, we show that the model can match the age profiles of key economic variables quite well. In particular, we are able to match the decline in labor supply that occurs in the 50s, as well as the hump-shaped profile of consumption. In both of these profiles, health and health investment play important roles. Much of the decline in labor supply is driven by declining health status. In addition, we show that much of the decline in consumption (on non-medical items) that occurs later in the life-course is driven by a rapid increase in medical expenditures that crowds out consumption on other items. Second, we decompose the Euler equation for health investment

to quantify the relative importance of the two motives for health investment. We show that the investment motive is about three times higher than the consumption motive during the early 20s, but decreases over the life-cycle until it disappears at retirement. In contrast, the consumption motive increases with age and surpasses the investment motive during the mid 40s. The driving force underlying the age-profile of the consumption motive is a decreasing marginal utility of health.

Our paper contributes to and bridges the gap between two literatures within economics. The first is the literature on the theory and, subsequently, the econometric estimation of models of health investment. The theoretical literature began with Grossman (1972a) but, since then, has grown substantially, with many authors such as Muurinen (1982) and Picone, Uribe and Wilson (1998) generalizing Grossman's original work. For a comprehensive discussion of these developments, we refer the reader to Grossman (1999). Accompanying these theoretical developments has been empirical work that has attempted to structurally estimate the parameters of Grossman's original model. While the later attempts by Wagstaff (1993) have proven more successful than the earlier attempts by Wagstaff (1986) and Grossman (1972b), no attempt has proven entirely satisfactory. We believe that the reason for this is that, as pointed out by Wagstaff (1993), previous attempts have largely relied on approximations of the Euler equation for health investment that do not adequately

account for the dynamics inherent in the health investment decision. By avoiding any linearizations of the Euler equations, our work avoids these complications.

Second, we also contribute to a growing literature that has incorporated health into computational models of life-cycle behavior. Many of these studies either incorporated health as an exogenous state variable (Rust and Phelan 1997; French 2005) or modeled health expenditure as exogenous shocks (Palumbo 1999; De Nardi, French and Jones 2006; Jeske and Kitao 2009). In contrast, our model endogenizes health investment, which allows us to answer the research questions proposed, and provides a more comprehensive analysis of the impact of health investment on relevant economic decisions.

This has spawned the most recent generation of papers that incorporates health into computational life-cycle models of behavior in which health is modeled as a durable consumption good *a la* Grossman (1972a). For example, Hall and Jones (2007) use a Grossman-type model to explain the recent increases in medical expenditures in the US. Yogo (2008) also builds a model of health investment to investigate the portfolio choice of retirees and argues that the large savings rate observed among the elderly is the consequence of a large bequest motive and not precautionary as others (*e.g.*, Palumbo 1999) have argued. Neither paper would have been able to make its conclusions without an endogenous health stock.

Our paper fits into this strand of the literature. However, there are notable differences. We investigate the *life-cycle* motives for health investment and its interaction with labor supply. While Hall and Jones (2007) investigate the evolution of medical expenses over time, they do not do so over the life-course nor do they consider labor supply. Although Yogo (2008) focuses on life-cycle behavior of retirees' consumption and portfolio choice with endogenous health investment, he does not consider labor supply. Finally, because we remain true to Grossman's original framework, we are also able to advance much of the literature on the estimation of models of health investment that was started by Michael Grossman and Adam Wagstaff.

The balance of this paper is organized as follows. Section 2 presents the model. Section 3 describes the life-cycle profiles of income, hours worked, medical expenditures and health status constructed from the PSID and the MEPS. Section 4 presents the parameterization of the model. Section 5 presents the life-cycle profiles generated from our benchmark model. Section 6 shows the decomposition of the consumption and investment motives. Section 7 conducts the sensitivity analysis for some key parameters in the model. Section 8 concludes.



## 2 Model

This section describes a life-cycle model with endogenous health accumulation. In this model, an individual lives at most  $J$  periods. For each age  $j \leq J$ , the conditional probability of surviving from age  $j - 1$  to  $j$  is denoted by  $\varphi_j \in (0, 1)$ . Notice that we have  $\varphi_0 = 1$  and  $\varphi_{J+1} = 0$ . The survival probability  $\{\varphi_j\}_{j=1}^J$  is treated as exogenously given.

### 2.1 Preferences

An individual derives utility from consumption, leisure and health. She maximizes expected discounted lifetime utility

$$E_0 \sum_{j=1}^J \beta^{j-1} \left[ \prod_{k=1}^j \varphi_k \right] U(c_j, l_j, h_j) \quad (1)$$

where  $\beta$  denotes the subjective discount factor,  $c$  consumption,  $l$  leisure, and  $h$  health status. The period utility function takes the form

$$U(c_j, l_j, h_j) = \frac{[\lambda(c_j^\rho l_j^{1-\rho})^\psi + (1-\lambda)h_j^\psi]^{\frac{1-\sigma}{\psi}}}{1-\sigma} \quad (2)$$

Motivated by the real business cycle literature such as Cooley and Prescott (1995), we assume that the elasticity of substitution between consumption and leisure is

one. The parameter  $\rho$  measures the weight of consumption. The elasticity of substitution between consumption and health is  $\frac{1}{1-\psi}$ . The parameter  $\lambda$  measures the relative importance of the consumption-leisure combination in the utility function. The parameter  $\sigma$  is the coefficient of relative risk aversion.

## 2.2 Budget Constraints

Each period this individual is endowed with one unit of non-sleeping time. She splits the time among working ( $n$ ), enjoying leisure ( $l$ ), and being sick( $s$ ). Therefore, we have the following time allocation equation

$$n_j + l_j + s_j = 1, \text{ for } 1 \leq j \leq J \quad (3)$$

Following Grossman (1972a), we assume sick time  $s_j$  is a decreasing function of health status

$$s_j = Qh_j^{-\gamma} \quad (4)$$

where  $Q$  is the scale factor and  $\gamma$  measures the sensitivity of sick time to health. Notice that in contrast to recent structural work that incorporates endogenous health accumulation (*e.g.*, Suen 2006, Feng 2008), in our model health does not directly affect labor productivity and/or survival probability. Allowing health to impact the

allocation of time but not labor productivity is consistent with Grossman (1972a), who says, “Health capital differs from other forms of human capital...a person’s stock of knowledge affects his market and non-market productivity, while his stock of health determines the total amount of time he can spend producing money earnings and commodities.”

This individual works until an exogenously given mandatory retirement age  $j_R$ . She differs in her labor productivity due to differences in age. We use  $\varepsilon_j$  to denote her efficiency unit at age  $j$ . Let  $w$  be the wage rate and  $r$  be the rate of return on asset holdings. Accordingly,  $w\varepsilon_j n_j$  is age- $j$  labor income. At age  $j$  she faces the following budget constraint

$$c_j + m_j + a_j \leq (1 - \tau_{ss})w\varepsilon_j n_j + (1 + r)a_{j-1}, \text{ for } j < j_R \quad (5)$$

where  $m_j$  is health investment in goods,  $a_j$  is asset holding, and  $\tau_{ss}$  is the Social Security tax rate.

Once the individual is retired, she receives Social Security benefits denoted by  $b$ . Following Imrohoroglu, Imrohoroglu, and Joines (1995), we model the Social Security system in a simple way. The Social Security benefits are calculated to be a fraction

$\kappa$  of some base income, which we take as the average lifetime labor income

$$b = \kappa \frac{\sum_{i=1}^{j_R-1} w \varepsilon_j n_j}{j_R - 1}.$$

$\kappa$  is referred to as the replacement ratio. The only role that government plays in this economy is to administer the Social Security system. An age- $j$  retiree faces the following budget constraint

$$c_j + m_j + a_j \leq b + (1 + r)a_{j-1} + T, \forall j \geq j_R \quad (6)$$

We assume that agents are not allowed to borrow so that

$$a_j \geq 0 \text{ for } 1 \leq j \leq J.$$

Finally, there is no annuity market.

## 2.3 Health Investment

Following Grossman (1972a), we assume that the individual has to invest in goods to produce health. The accumulation of health across ages is given by

$$h_{j+1} = (1 - \delta_{h_j})h_j + Bm^\xi \quad (7)$$

where  $\delta_{h_j}$  is the age-dependent depreciation rate of health stock,  $B$  measures the productivity of medical care technology, and  $\xi$  represents the return to scale for health investment.

We assume that the age-dependent depreciation rate of health stock  $\delta_{h_j}$  takes the form

$$\delta_{h_j} = \frac{\exp(a_0 + a_1j + a_2j^2)}{1 + \exp(a_0 + a_1j + a_2j^2)}. \quad (8)$$

This functional form guarantees that  $\delta_{h_j} \in (0, 1)$  and (given suitable values for  $a_1$  and  $a_2$ ) increases as the individual ages.

## 2.4 Individual's Problem

At age  $j$ , this individual solves a dynamic programming problem. The state space at the beginning of age  $j$  is described by a vector  $(a_{j-1}, h_j)$ , where  $a_{j-1}$  is the asset holding at the beginning of age  $j$ , and  $h_j$  is health status at age  $j$ . Let  $V_j(a_{j-1}, h_j)$

denote the value function at age  $j$  given the state vector  $(a_{j-1}, h_j)$ . The Bellman equation is then given by

$$V_j(a_{j-1}, h_j) = \max_{c_j, m_j, a_j, l_j, n_j} \{U(c_j, l_j, h_j) + \beta \varphi_{j+1} E_j V_{j+1}(a_j, h_{j+1})\} \quad (9)$$

subject to

$$c_j + m_j + a_j \leq (1 - \tau_{ss})w\varepsilon_j n_j + (1 + r)a_{j-1}, \forall j < j_R$$

$$c_j + m_j + a_j \leq b + (1 + r)a_{j-1}, \forall j_R \leq j \leq J$$

$$h_{j+1} = (1 - \delta_{h_j})h_j + Bm_j^\xi, \forall j$$

$$n_j + l_j + s_j = 1, \forall j$$

$$a_j \geq 0, \forall j$$

and the usual non-negativity constraints.

### 3 The Data

We employ data from two sources. The first is the Panel Study of Income Dynamics (PSID), which we use to construct life-cycle profiles for income, hours worked and health status. The second is the Medical Expenditure Survey (MEPS), which we

use to construct life-cycle profiles for medical expenditures.

### **3.1 Panel Study of Income Dynamics**

Our PSID sample spans the years 1968 to 2005. The PSID contains an over-sample of economically disadvantaged people called the Survey of Economic Opportunities (SEO). We follow Lillard and Willis (1978) and drop the SEO due to endogenous selection. Doing this also makes the data more nationally representative. Our labor income measure includes any income from farms, businesses, wages, roomers, bonuses, overtime, commissions, professional practice and market gardening. This is the same income measure used by Meghir and Pistaferri (2004). Our measure of hours worked is the total number of hours worked in the entire year. Our health status measure is a self-reported categorical variable in which the respondent reports that her health is in one of five states: excellent, very good, good, fair, or poor. While these data can be criticized as being subjective, Smith (2003) and Baker, Stabile and Deri (2004) have shown that they are strongly correlated with both morbidity and mortality. In addition, Bound (1991) has shown that they hold up quite well against other health measures in analyses of retirement behavior. Finally, in a quantitative study of life-cycle behavior such as this, they have the desirable quality that they change over the life-course and that they succinctly summarize morbidity. A battery

of indicators of specific medical conditions such as arthritis, diabetes, heart disease, hypertension, etc. would not do this. For the purposes of this study, we map the health variable into a binary variable in which a person is either healthy (self-rated health is either excellent, very good or good) or a person is unhealthy (self-rated health is either fair or poor). This is the standard way of partitioning this health variable in the literature.

Figures 1 through 3 show the life-cycle profile of income, hours and health. These calculations were made by estimating linear fixed effects regressions of the outcomes on a set of age dummies on the sub-sample of men between ages 25 and 75. Because we estimated the individual fixed effects, our estimates are not tainted by heterogeneity across individuals (and, by implication, cohorts). Each figure plots the estimated coefficients on the dummy variables. Figure 1 shows the income profile (in 2004 dollars). The figure shows a hump-shape with a peak at about 60K in the early 50s. A major source of this decline is early retirements. This can be seen in Figure 2, which plots yearly hours worked. Hours worked are pretty steady at just over 40 per week until about the mid 50s when they start to decline quite rapidly. Figure 3 shows the profile of health status. The figure shows a steady decline in health. Approximately 95% of the population reported being healthy at age 25, and this declined to just under 60% at age 75.



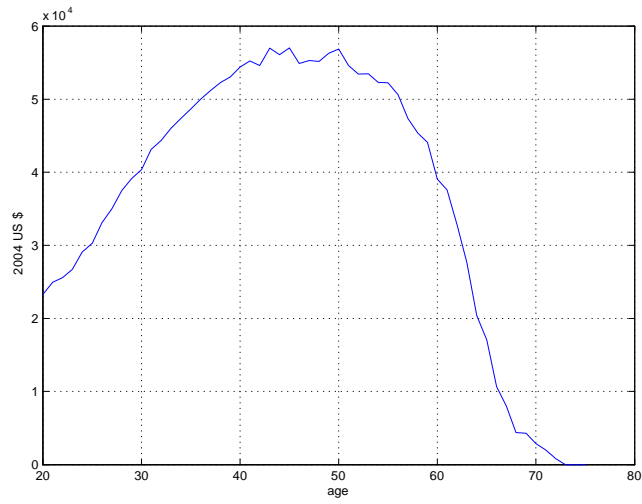


Figure 1: Life-cycle profile of labor income: PSID data

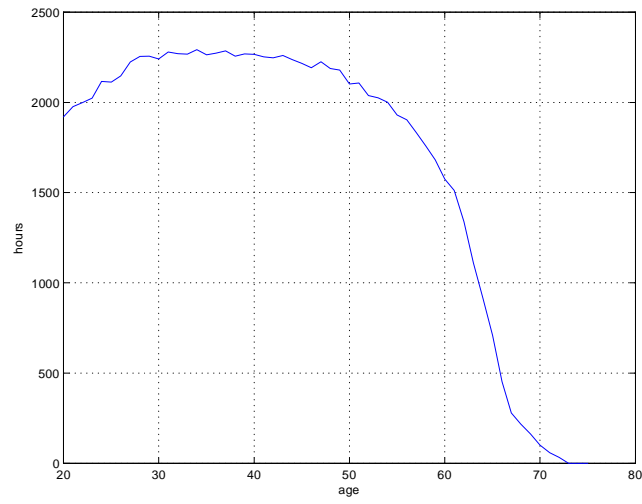


Figure 2: Life-cycle profile of working hours: PSID data

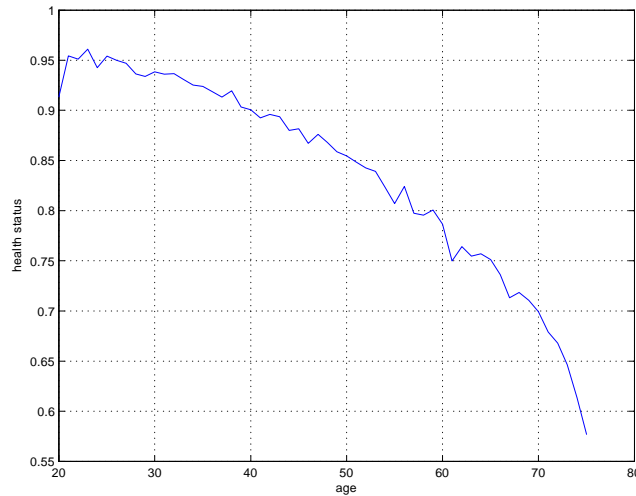


Figure 3: Life-cycle profile of health status: PSID data

### 3.2 Medical Expenditure Survey

Our MEPS sample spans the years 2003-2006. As discussed in Kashihara and Carper (2008), the MEPS measure of medical expenditures we employ includes “direct payments from all sources to hospitals, physicians, other health care providers (including dental care) and pharmacies for services reported by respondents in the MEPS-HC.” Note that these expenditures include both out-of-pocket expenditures and expenditures from the insurance company.

Figure 4 shows the life-cycle profile of medical expenditures (in 2004 dollars). The top profile was calculated in the same way as the profiles in the three previous figures. The bottom profile was calculated using a quantile regression. Accordingly,

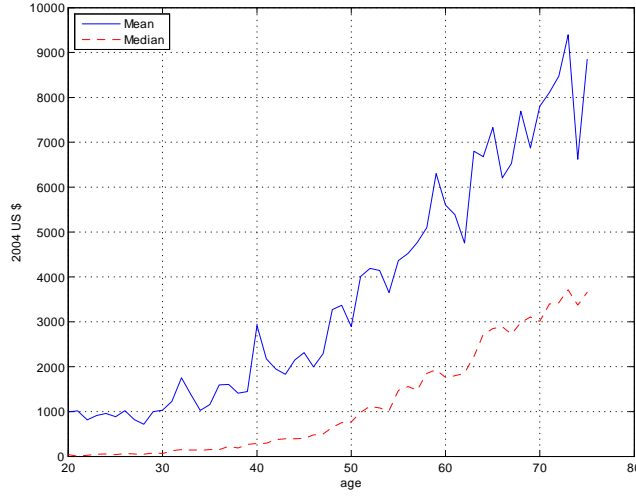


Figure 4: Life-cycle profile of medical expenditures: MEPS data

the top figure reports the means and the bottom figure reports the median by age. Both profiles show an increasing and convex relationship with age. Perhaps not surprisingly, we see that the medians are substantially below the means. This is almost certainly the consequence of the notoriously fat tail in medical expenditure data. Because we have a representative agent model, we will be matching the mean profile. However, the divergence between the medians and the means underscores the need to incorporate heterogeneity into the existing framework in future research.

## 4 Calibration

We now outline the calibration of the model's parameters. For the parameters that are commonly used, we borrow from the literature. For those that are model-specific, we choose parameter values to minimize the distance between the labor income profiles in the model and the data.<sup>1</sup>

### 4.1 Demographics

The model period is five years. An individual is assumed to be born at the real-time age of 20. Therefore, the model period  $j = 1$  corresponds to ages 20-24,  $j = 2$  corresponds to ages 25-29, and so on. Death is certain after age  $J = 16$ , which corresponds to ages 95-99. The conditional survival probabilities  $\{\varphi_j\}_{j=1}^J$  are taken from the US Life Tables 2002. Retirement is mandatory and occurs at age  $j_R = 10$ , which corresponds to ages 65-69. We take the age-efficiency profile  $\{\varepsilon_j\}_{j=1}^{j_R-1}$  from Conesa, Kitao and Krueger (2009), who construct it following Hansen (1993).

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<sup>1</sup>The reason we choose to match the life-cycle labor income profile is that it is the least health-related among the other life-cycle profiles we want to study. In other words, we want to evaluate the performance of the model on the health and health investment, and we also want to analyze the interaction among health investment, consumption and labor supply. This consideration narrows our choice of targets for calibration.

## 4.2 Preferences

We set the annual subjective time discount factor to be 0.971, which is in the range of widely used values in the literature. Therefore,  $\beta = (0.971)^5$ . We choose a coefficient of relative risk aversion  $\sigma = 2$ , which is also a value widely used in the literature (e.g., Imrohoroglu, Imrohoroglu, and Joines (1995); Fernandez-Villaverde and Krueger (2002)). Following Yogo (2008), we set the elasticity of substitution between consumption and health to be  $\frac{1}{1-\psi} = 0.11$ ; this implies  $\psi = -8$ . Since the elasticity of substitution is near its lower bound of zero (which corresponds to the extreme case of Leontief preferences), health and consumption are complements. Since this is a key parameter, we will conduct a sensitivity analysis later.

## 4.3 Social Security

We set the Social Security tax rate to be 10.6%, which is the current rate for U.S. Old-Age and Survivors Insurance (OASI). The Social Security replacement ratio  $\kappa$  is set to be 30%.<sup>2</sup>

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<sup>2</sup>Imrohoroglu, Imrohoroglu and Joines (1995) find that the optimal Social Security replacement ratio is 30%.

## 4.4 Factor Prices

The wage rate  $w$  is a normalization, which we set at 0.80. Since it is a normalization, changing it to other values does not alter the results. We set the interest rate at 4%.

## 4.5 The Remaining Parameters

There are nine model-specific parameters that remain: the weight of consumption in this consumption-leisure combination ( $\rho$ ), the share of consumption-leisure composition in utility function ( $\lambda$ ), the productivity of health accumulation technology ( $B$ ), the return to scale for health investment ( $\xi$ ), the scale factor of sick time ( $Q$ ), the elasticity of sick time to health ( $\gamma$ ), and three parameters that determine the age-dependent depreciation rate of health stock ( $a_0, a_1, a_2$ ). Our strategy is to choose these parameter values so that the model can replicate, as close as possible, the life-cycle disposable labor income profile for working age (ages 20-64) people in the data. We take the minimum squared error (MSE) as the measurement of distance

$$\min \sum_{j=1}^{j_R-1} \left( \frac{w(1 - \tau_{ss})\varepsilon_j n_j - income\_data_j}{w(1 - \tau_{ss})\varepsilon_j n_j} \right)^2$$

Since the model-generated labor income is on a different scale than in the data, we normalize the first period (ages 20-24) labor income data to match that in the

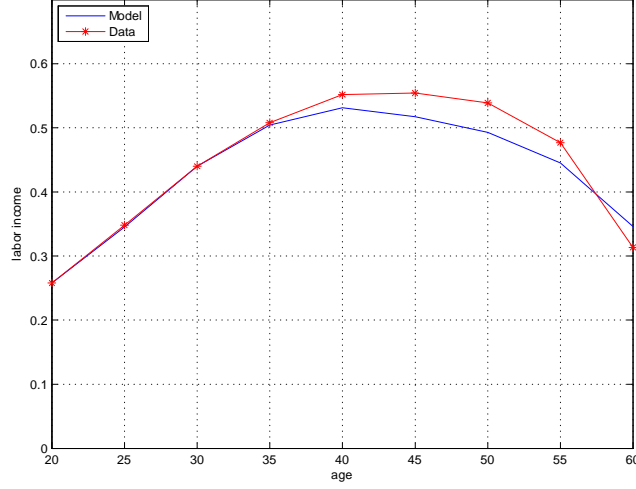


Figure 5: Life-cycle profile of labor income: model vs. data

model. Figure 5 shows the life-cycle profile of disposable labor income in both the model and the data. The model matches the data very well. Particularly, the model replicates the data almost perfectly in the first four periods. The deviation of the model from the data is only 2.95% over all nine periods. We summarize our baseline parameterization in Table 1.

## 5 Results

Using the parameter values from Table 1, we compute the model using standard numerical methods. We report model-generated life-cycle profiles in Figure 6 to Figure 10.

| Parameter                         | Description                              | Value                | Source                    |
|-----------------------------------|--|----------------------|---------------------------|
| Demographics                      |  |                      |                           |
| $J$                               | maximum life span                        | 16 (95 – 99)         | US Life Table 2002        |
| $j_R$                             | mandatory retirement age                 | 10 (65 – 69)         |                           |
| $\{\varphi_j\}_{j=1}^J$           | conditional survival probabilities       | Data                 |                           |
| Preferences                       |  |                      |                           |
| $\beta$                           | subjective discount factor               | (0.971) <sup>5</sup> | Yogo (2008)               |
| $\sigma$                          | CRRA coefficient                         | 2                    |                           |
| $\psi$                            | elasticity b/w cons. and health          | −8                   |                           |
| $\rho$                            | share of $c$ in $c$ -leisure combination | 0.34                 |                           |
| $\lambda$                         | share of cons-leisure com. in utility    | 0.80                 |                           |
| Health Accumulation               |  |                      |                           |
| $a_0$                             | dep. rate of health                      | −4.00                | calibrated                |
| $a_1$                             | dep. rate of health                      | 0.05                 | calibrated                |
| $a_2$                             | dep. rate of health                      | 0.00032              | calibrated                |
| $B$                               | productivity of health technology        | 0.95                 | calibrated                |
| $\xi$                             | return to scale for health investment    | 0.8                  | calibrated                |
| Sick Time                         |  |                      |                           |
| $Q$                               | scale factor of sick time                | 0.07                 | calibrated                |
| $\gamma$                          | elasticity of sick time to health        | 1.5                  | calibrated                |
| Labor Productivity                |  |                      |                           |
| $\{\varepsilon_j\}_{j=1}^{j_R-1}$ | age-efficiency profile                   |                      | Conesa et al. (2009)      |
| Social Security                   |  |                      |                           |
| $\tau_{ss}$                       | Social Security tax rate                 | 10.6%                | Data                      |
| $\kappa$                          | Social Security replacement ratio        | 30%                  | Imrohoroglu et al. (1995) |
| Factor Prices                     |  |                      |                           |
| $w$                               | wage rate                                | 0.80                 |                           |
| $r$                               | interest rate                            | 4%                   |                           |

Table 1: Parameters of the model



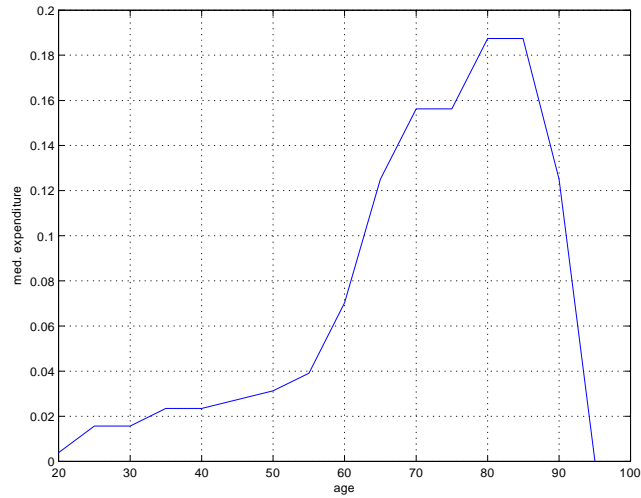


Figure 6: Life-cycle profile of medical expenditure: model

Figure 6 shows the life-cycle profile of health investment ( $m$ ). An interesting pattern emerges. Health investment increases steadily until the mid 50s, at which point it accelerates. From ages 55-59 to ages 80-85, it increases dramatically from 0.039 to 0.188 - a five-fold increase. However, after ages 85-89, the model predicts a sharp decline in medical spending. This is a consequence of the assumption of certain death after age 100 in the model. A forward-looking individual knows that she will not need any health investment after age 100; therefore, she begins to disinvest in health as the death date approaches.

Figure 7 shows the life-cycle pattern of health expenditure-labor income ratio. In the data, this ratio is very low and stable until age 50, then it increases dramatically

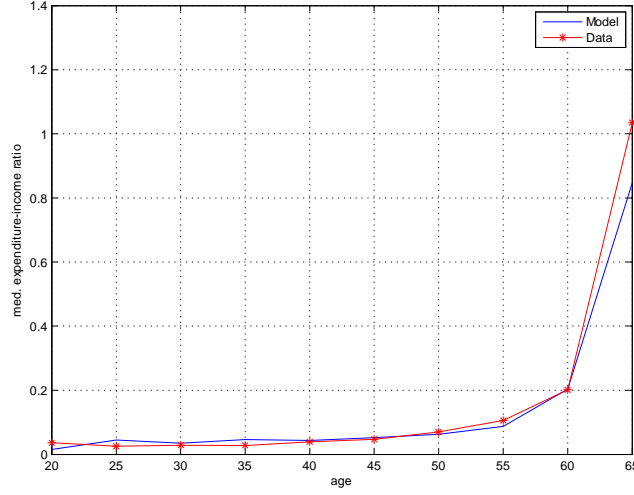


Figure 7: Life-cycle profile of medical expenditure-income ratio: model vs. data

after age 55. The model captures this pattern very well. From ages 55-59 to ages 65-69, this ratio increases from 0.11 to 1.03 in the data, while the model predicts that the health expenditure-labor income ratio increases from 0.09 to 0.84.

Health investment (in conjunction with depreciation) determines the evolution of the health stock. Figure 8 displays the life-cycle profile of health. The model can produce decreasing health status over the life-cycle. However, in the model, as shown in Figure 7, an individual tends to invest (relative to her labor income) more than she does in the data from age 25 to age 45. Thus, the health stock is higher than it is in the data. In contrast, the under-investment at the later age in the model induces lower health stock compared to the data.

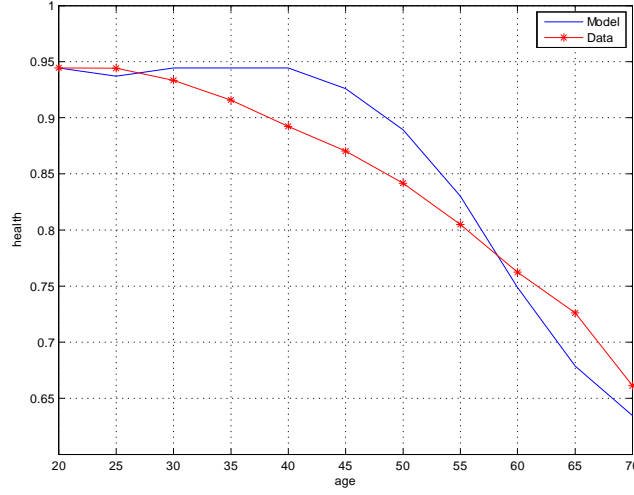


Figure 8: Life-cycle profile of health status: model

The model also does well in replicating other economic decisions over the life-cycle. Figure 9 shows the life-cycle profile of working hours. The model captures the hump-shape of working hours. In the data, individuals devote about 34% of their non-sleeping time to working at age 20-24. The fraction of working time increases to its peak at ages 35-39, and it is quite stable until ages 45-59. It then decreases sharply from about 38% at ages 45-49 to 22% at ages 60-64. In the model, the fraction of working hours reaches the peak (about 38%) at ages 40-45. It then decreases by 11% to about 27% at ages 60-64. The health stock plays an important role in the declining portion of the working hours profile; as health status declines, sick time increases over the life-cycle, which, in turn, encroaches upon a person's ability to

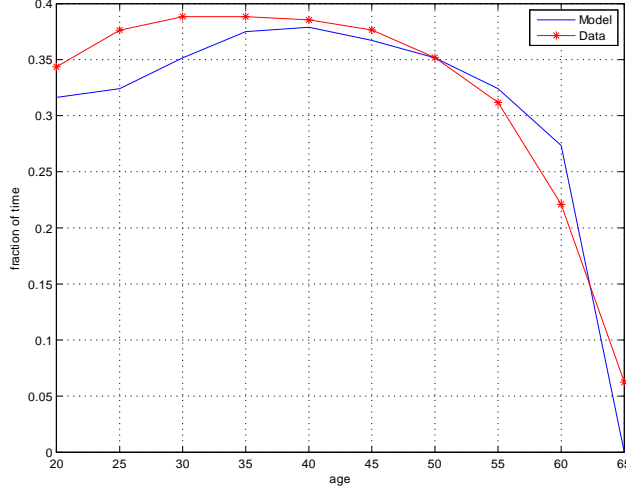


Figure 9: Life-cycle profile of working hours: model

work. Our model predicts that from ages 45-49 to 60-64, the fraction of sick time in the non-sleeping time increases from 7.85% to 10.80%, which accounts for about 28% of the decline in working hours in the model.

Figure 10 shows the life-cycle profile of consumption (excluding medical expenditure) in the model. It also exhibits a hump-shape. Indeed, consumption declines dramatically after ages 60-64 which is exactly when medical expenditure increases precipitously. From ages 60-64 to ages 65-69, non-medical consumption decreases by 22% from 0.406 to 0.317, while medical expenditure increases by about 80%, from 0.070 to 0.125. When we combine non-medical consumption and medical expenditure, as in Figure 11, we see a smoother profile. The main message that we obtain

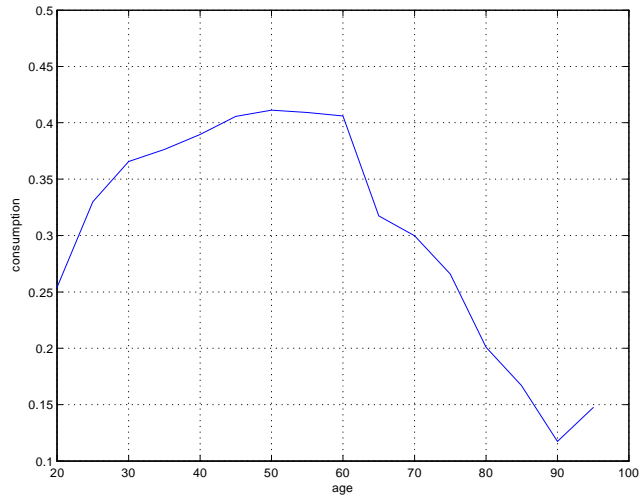


Figure 10: Life-cycle profile of consumption: model

from these two graphs is that health investment “crowds out” consumption at later ages.

Finally, Figure 12 shows the time allocation between leisure and sick time. At ages 20-24, leisure accounts for about 60% of non-sleeping time. Due to the hump-shape of working hours (see Figure 9), it gradually decreases to around 54% at ages 40-45 after which it steadily increases to 62% at ages 60-64. Due to retirement at age 65, leisure increases dramatically to 87% of non-sleeping time at ages 65-69. After age 70, it begins to decrease again as sick time accelerates and starts to dominate. Finally, at the end of the life-course, sick time accounts for more than half of non-sleeping time.

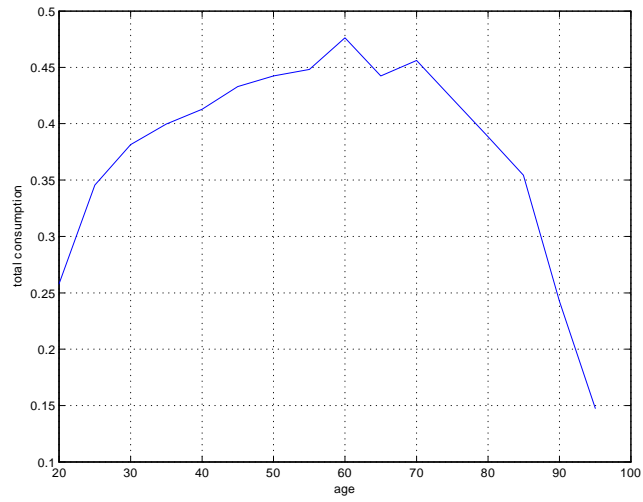


Figure 11: Life-cycle profile of total consumption: model

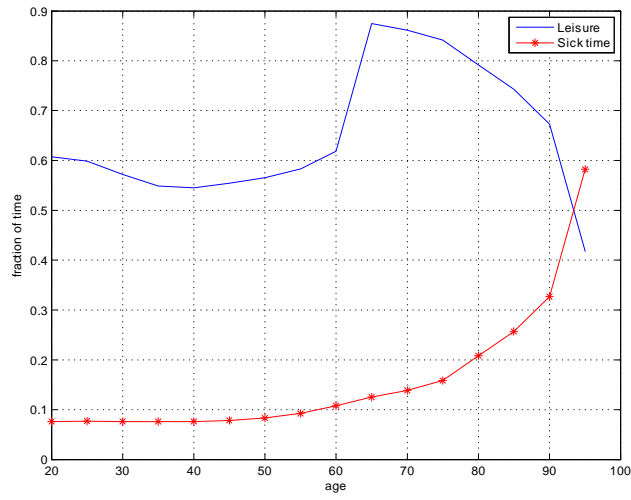


Figure 12: Life-cycle profile of sick time and leisure: model

To summarize, our life-cycle model with endogenous health accumulation is able to replicate life-cycle profiles from the MEPS and the PSID. First, it captures the hump-shape of total consumption. Second, it captures the hump-shape of working hours. Third, and probably the most important, it captures the rising medical expenditure-labor income ratio.

## 6 Motives for Health Investment

Based on the success of the model, we would like to use the model to quantify the relative importance of the consumption and investment motives for health investment. To do this, we begin with the Euler equation for health investment:<sup>3</sup>

$$\frac{\partial U}{\partial c_j} = \beta \varphi_{j+1} MPM_j \left[ \frac{\partial U}{\partial h_{j+1}} - \mathcal{I}_j \frac{\partial U}{\partial c_{j+1}} w(1 - \tau_{ss}) \varepsilon_{j+1} \frac{\partial s_{j+1}}{\partial h_{j+1}} + (1 - \delta_{h_{j+1}}) \frac{\partial U / \partial c_{j+1}}{MPM_{j+1}} \right]$$

$$\text{where } \mathcal{I}_j \begin{cases} = 1 & \text{if } j < j_R \\ = 0 & \text{otherwise} \end{cases} \quad (10)$$

where  $MPM_j = B\xi m_j^{\xi-1}$  is the marginal product of health investment at age  $j$ .

This equation provides the optimal rule for health investment. The marginal utility of consumption at age  $j$ , on the left-hand side of the equation, represents the marginal

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<sup>3</sup>Please refer to Appendix 1 for the derivation of this equation.

cost of investing one additional unit of goods in health accumulation. The marginal benefits on the right-hand side consist of three terms. The first term,  $MPM_j \frac{\partial U}{\partial h_{j+1}}$ , shows that improvements in health due to investment will directly increase utility. This is the “consumption” motive for health investment. Better health tomorrow will also raise labor income *via* a reduction in sick time. This is the “investment” motive for the health investment and is captured by the second term on the right-hand side of equation (10),  $-MPM_j \frac{\partial U}{\partial c_{j+1}} w \varepsilon_{j+1} \frac{\partial s_{j+1}}{\partial h_{j+1}}$ . Note that since  $\varepsilon_j = 0$  for age  $j \geq j_R$ , the “investment” motive disappears after retirement. Also, note that better health tomorrow provides a more favorable starting point for future health accumulation. This is shown in the third term  $(1 - \delta_{h_{j+1}}) \frac{MPM_j}{MPM_{j+1}} \frac{\partial U}{\partial c_{j+1}}$  on the right-hand side. This is the “continuation value” of health investment.

Figure 13 shows a decomposition of these three terms over the life-cycle. Top graph in this figure shows that the consumption motive, which is driven by the marginal utility of health, increases with age as the health stock decreases. In middle graph, the investment motive exhibits an interesting “U” shape and disappears after retirement. The “U” shape is a result of an interaction of three factors: from ages 20-24 to 60-64, marginal utility of consumption is decreasing ( $\frac{\partial U}{\partial c_{j+1}}$ ); the age-efficiency profile ( $\varepsilon_{j+1}$ ) exhibits hump-shape; and the marginal benefit of reducing sick time via better health ( $\frac{\partial s_{j+1}}{\partial h_{j+1}}$ ) is increasing. The continuation value in bottom graph also



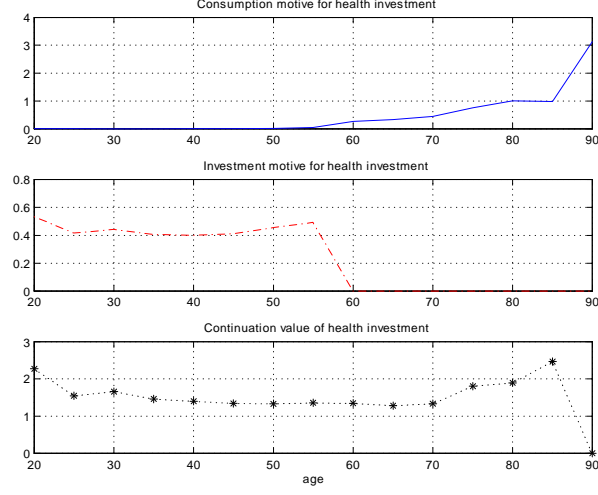


Figure 13: Decomposition of Euler equation for health

shows “U” shape which is mainly affected by the marginal utility of consumption.

Noting that the continuation value term contains the present value of future consumption and investment motives, we can further decompose health investment motives by repeatedly substituting out this term and obtain:

$$\frac{\partial U}{\partial c_j} = MPM_j \sum_{t=1}^{J-j} \underbrace{\beta^t \left( \prod_{k=j+1}^{j+t} \varphi_k \right) \left( \prod_{k=j+1}^{j+t-1} (1 - \delta_{h_k}) \right)}_{\text{effective discount factor}} \left[ \underbrace{\frac{\partial U}{\partial h_{j+t}}}_{\text{consumption motive}} - \underbrace{\frac{\partial U}{\partial c_{j+t}} w \varepsilon_{j+t} \frac{\partial s_{j+t}}{\partial h_{j+t}}}_{\text{investment motive}} \right]. \quad (11)$$

This equation states that the marginal benefit of one additional unit of goods investment in health at age  $j$  is the sum of the discounted accumulative “consumption” and “investment” values from age  $j + 1$  to the end of life  $J$ . The effective discount

factor for  $t$  periods ahead of age  $j$  consists of three components: the subjective time discount factor  $\beta^t$ , unconditional survival probability up to age  $j + t$ , and the accumulation depreciation rate from age  $j$  to age  $j + t$ .

We show the accumulative “consumption” and “investment” motives in Figure 14. In this figure, we see that the accumulative “investment” motives decrease over the life-cycle and disappear after retirement. In contrast, the accumulative “consumption” motive increases with age. For the young, who are very healthy, the marginal utility of health is extremely small. On the other hand, they are very active in the labor market, and, therefore, the benefits from working longer hours by reducing sick time are important. Accordingly, the investment motive dominates the consumption motive at this point in the life-cycle. Indeed, during the early 20s, the investment motive is about three times higher than consumption motive. However, as people age, their health deteriorates and the marginal utility of health increases. Meanwhile, they face a shorter working life as they near retirement. Consequently, the consumption motive surpasses the investment motive during the mid 40s. As an individual enters her retirement, the investment motive disappears. Thus, her health investment is affected only by the consumption motive, which in turn is mainly driven by the rising marginal utility of health.

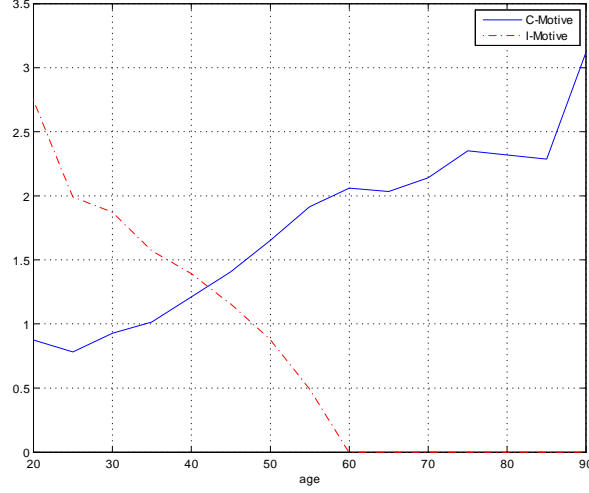


Figure 14: Decomposition of consumption and investment motive

## 7 Sensitivity Analysis

### 7.1 No Health Investment

As a counter-factual experiment, we would like to see what would happen if we shut down health investment completely by setting  $B = 0$ . In Figure 15, we show the life-cycle profiles of labor income, consumption, working hours and health status of the model when there is no health investment. We see that labor income in top left graph is significantly lower than that in the benchmark model. The main reason is that, as shown in bottom left, working hours are much lower than in the benchmark model. Since there is no health investment to compensate for the loss of

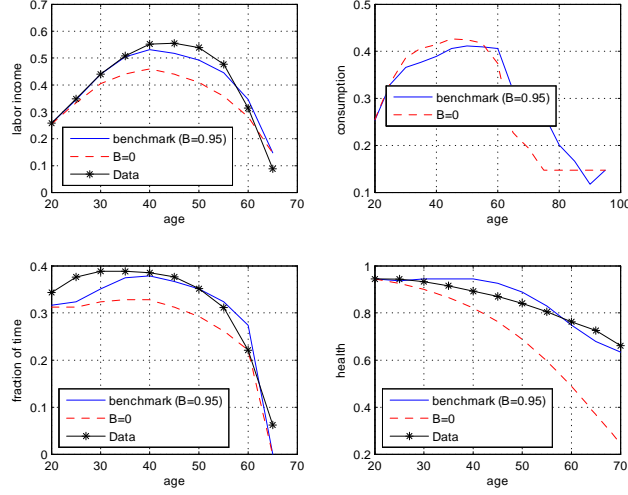


Figure 15: Life-cycle profiles of the model without health investment

health stock, health status is purely driven by natural depreciation. Bottom right shows that health status in the model decreases much faster than that in the data. As a function of health status, sick time thus increases much faster and crowds out working time more severely. Lower health status also reduces consumption later in life. This is partly due to having lower income but also to the complementarity between health and consumption.

## 7.2 No Depreciation of Health

Figure 16 shows the life-cycle profiles of the model without depreciation of health stock, *i.e.*,  $\delta_{h_j} = 0, \forall j$ . In this scenario, an individual will optimally choose a fixed

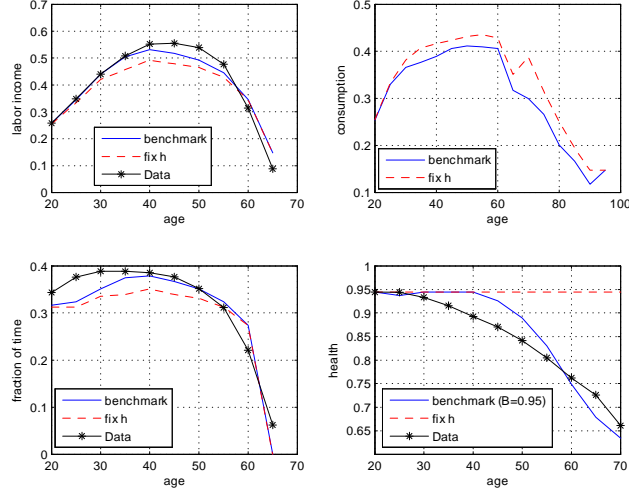


Figure 16: Life-cycle profiles of the model without depreciation of health

health stock that does not change over the life-cycle. Since health is complementary to both consumption and leisure, better health induces higher consumption and leisure throughout the life-cycle. Higher leisure crowds out working time, and so, working hours are lower than in the benchmark model.

### 7.3 Elasticity of Health

The parameter  $\psi$  determines the elasticity of substitution between health and consumption. The benchmark value that we chose ( $\psi = -8$ ) implies that health is complementary to both consumption and leisure. To assess sensitivity to this parameter, we investigate what happens when  $\psi = -1$  and  $\psi = 0$ . The implied elasticities of

substitution associated with these parameters are 0.5 and 1 (the utility function thus is Cobb-Douglas), respectively. Note that as  $\psi$  increases, health and consumption become more substitutable.

Figure 17 shows the life-cycle profiles in each of the three cases. We see that as health becomes substitutable with consumption, it does not change the shape of any of the life-cycle profiles, but it does change the shape of the accumulative consumption motive over the life-cycle. Indeed, in Figure 18 we see that the consumption motive in the cases of  $\psi = -1$  and  $\psi = 0$  decreases over the life-cycle. The reason is that as consumption decreases during old age, health decreases less than in the benchmark case, since health is now a substitute for consumption. We can see this in middle right graph in Figure 17. Therefore, the marginal utility of health later in the life-cycle under both cases is much lower than that in the benchmark case. The effective discount factor in equation (11) dominates, which causes the accumulative consumption motive to decrease over the life-cycle.

## 7.4 Borrowing Constraints

In the benchmark model, we do not allow the individual to borrow. Here, we relax this assumption. Figure 19 shows that without a borrowing constraint, an individual borrows at the beginning of the life-cycle. She pays off all debts during the mid 30s

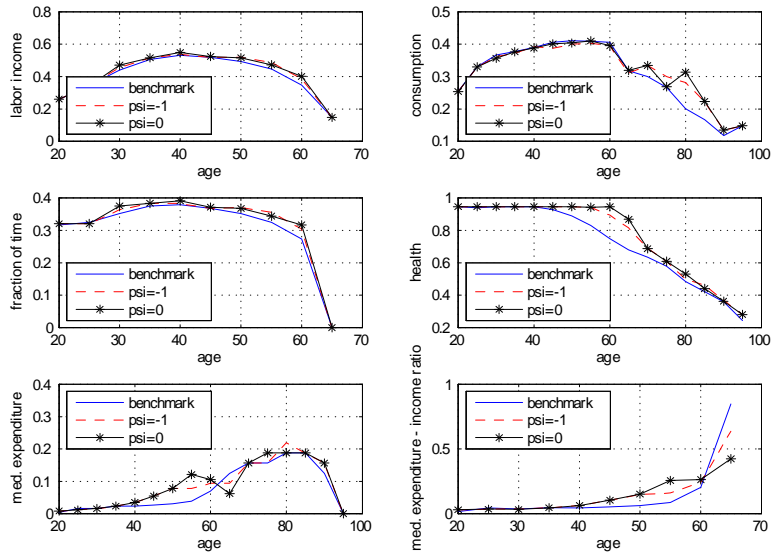


Figure 17: Life-cycle profiles of different elasticities of health

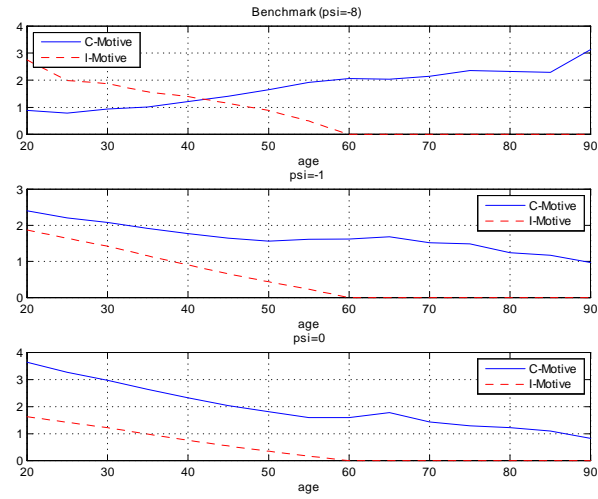


Figure 18: Decomposition of accumulative consumption and investment motive: sensitivity on  $\psi$

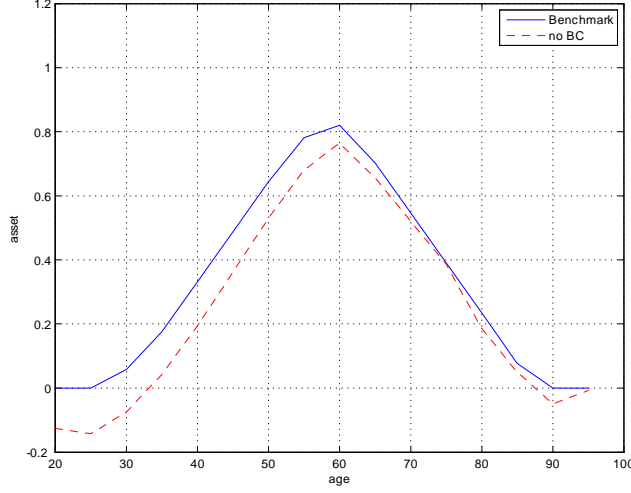


Figure 19: Life-cycle profile of asset accumulation: borrowing constraint vs. non borrowing constraint

and then begins to accumulate positive assets. Assets reach their peak just before retirement. Towards the end of the life-cycle, the consumer dissaves to smooth consumption. Compared to the benchmark case with a borrowing constraint, asset holdings are lower at all ages.

The absence of a borrowing constraint does not significantly affect the life-cycle-profiles. Figure 20 shows that the most significant effect is on working hours. Since an individual can borrow to smooth consumption, she does not need to work as hard during the early stage of the life-cycle. Working hours during the 20s are lower than in the benchmark case. However, since she has to pay back debt, working hours increase during mid-age. The borrowing ability does help in smoothing consumption,



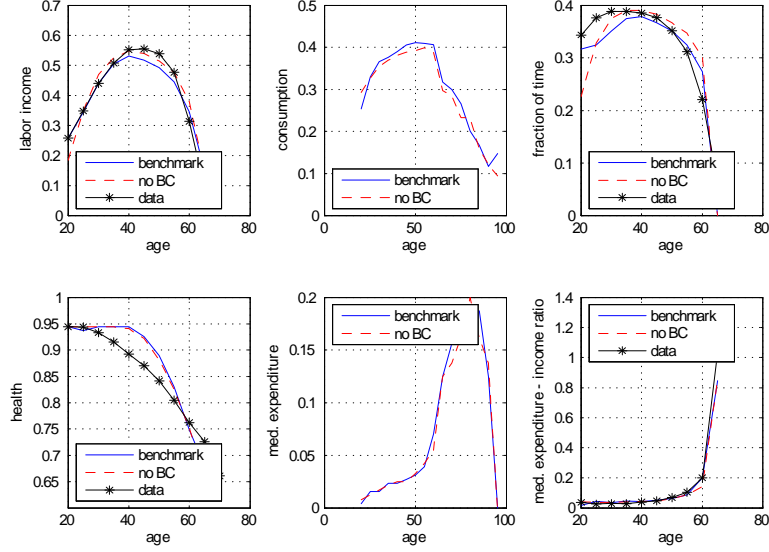


Figure 20: Life-cycle profiles of the economy with and without borrowing constraint

as in top middle graph of Figure 20. However, it does not affect the life-cycle profiles of health status, health investment and the medical expenditure-labor income ratio too much.

Allowing an individual to borrow freely also does not affect the consumption and investment motives significantly except during the last three periods. This is because the last period consumption in the “no borrowing constraint” case is substantially lower than that in the benchmark case. This reduces the marginal utility of health (since  $\frac{\partial(\partial u/\partial h)}{\partial c} > 0$ ), which is a major term to determine the consumption motive.

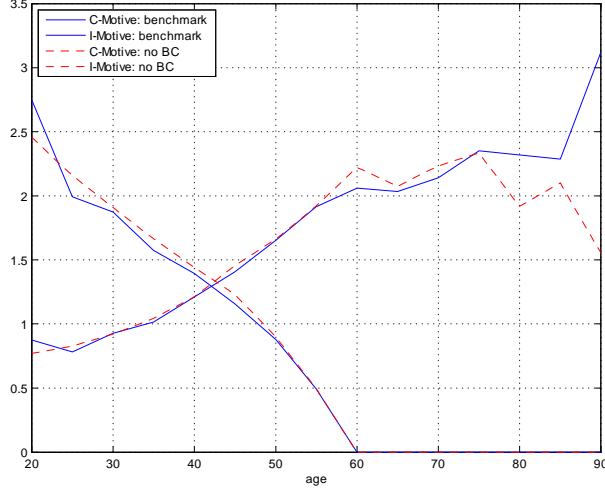


Figure 21: Decomposition of consumption and investment motive: borrowing constraint vs. non-borrowing constraint

## 8 Conclusions

We studied the motives underlying the life-cycle behavior of health investment. To accomplish this, we calibrated a standard model of health investment to match the life-cycle profile of labor income in the data. We found that the calibrated model fits key life-cycle profiles of consumption, working hours, health status and the medical expenditure-labor income ratio very well. We then used the Euler equation for health investment to decompose the motives for health investment into their consumption and investment values. We found that the investment motive is about three times higher than the consumption motive during the early 20s. It then steadily declines

with age until retirement, when it is exactly zero. In contrast, the consumption motive increases with age due to an increasing marginal utility of health. It surpasses the investment motive during mid 40s.

Our model can be extended along several dimensions. First, we assume an exogenous survival probability for the sake of computational simplicity. However, health investment should affect survival probabilities. By allowing for an endogenous survival probability, we would incorporate another benefit of health investment. In the Euler equation (10), this implies that health investment not only has “consumption” and “investment” motives, but it also increases the effective discount factor by raising  $\varphi_{j+1}$ . Second, we assume mandatory retirement at age 65 in the model. However, in the data we do see some early retirements. How does health status affect an individual’s retirement decision? Extension of the model to make retirement an endogenous decision would shed light on this question. Finally, we do not have uncertainty over health status in the model. Adding uncertainty would allow us to analyze the effect of health insurance (public or private) on individuals’ health investment. It will also help us to better understand the distribution of health expenditures as we mentioned in the data section.

With these extensions, this model provides a platform to carry out some very important policy experiments. For example, we can analyze the welfare cost of the

Medicare system. The benefits of Medicare arise from facilitating risk-sharing. However, Medicare has costs. First, the Medicare tax distorts labor supply. Second, if individuals know that they will be insured against health risk when they are older, they may reduce their health investment when young, which, in turn, reduces average health status, which, thus, incurs higher medical costs for society. Another interesting policy experiment is to analyze the welfare gain (or loss) of a change from the current system in the United States, which contains both employer-provided health insurance along with public health insurance (such as Medicare and Medicaid) to an alternative regime such as universal health care. Finally, one can also use this framework to quantify the effects of tax-favorable health savings accounts (HSAs) on savings, consumption and health investment. In this sense, we view this paper as a first step in a more ambitious research agenda.

## References

- [1] Baker, M., M. Stabile and C. Deri (2004): “What Do Self-Reported, Objective Measures of Health Measure?” *Journal of Human Resources*, 39, 1067-1093.
- [2] Bound, J. (1991): “Self-Reported Versus Objective Measures of Health in Retirement Models,” *Journal of Human Resources*, 26, 106-138.

- [3] Carroll, C.D. (1997): “Buffer Stock Saving and the Life-Cycle/Permanent Income Hypothesis,” *Quarterly Journal of Economics*, 107, 1-56.
- [4] Conesa, J., S. Kitao and D. Krueger (2009): “Taxing Capital Income? Not a Bad Idea After All!” *American Economic Review*, 99, 25-48.
- [5] Cooley, T. and E. C. Prescott (1995): “Economic Growth and Business Cycles,” in *Frontiers of Business Cycle Research*, ed. by Thomas Cooley, Princeton University Press.
- [6] De Nardi, M., E. French and J. B. Jones (2006): “Differential Mortality, Uncertain Medical Expenses, and the Saving of Elderly Singles,” unpublished mimeo.
- [7] Deaton, A. (1992): *Understanding Consumption*. Oxford: Clarendon Press.
- [8] Feng, Z. (2008): “Macroeconomic Consequences of Alternative Reforms to the Health Insurance System in the U.S.,” University of Miami, unpublished mimeo.
- [9] Fernandez-Villaverde, J. and D. Krueger (2002): “Consumption and Saving over the Life Cycle: How Important are Consumer Durables?” *Proceedings of the 2002 North American Summer Meetings of the Econometric Society: Macroeconomic Theory*.

- [10] French, E. (2005): “The Effects of Health, Wealth and Wages on Labor Supply and Retirement Behavior,” *Review of Economic Studies*, 72, 395-428.
- [11] Gorinchas, P.O. and J.A. Parker (2002): “Consumption over the Life-Cycle,” *Econometrica*, 70, 47-89.
- [12] Grossman, M. (1972a): “On the Concept of Health Capital and the Demand for Health,” *Journal of Political Economy*, 80, 223-255.
- [13] Grossman, M. (1972b): *The Demand for Health: A Theoretical and Empirical Investigation*, New York: NBER Press.
- [14] Grossman, M. (1999): “The Human Capital Model of the Demand for Health,” NBER Working Paper 7078.
- [15] Hall, R.E. and C.I. Jones (2007): “The Value of Life and the Rise in Health Spending,” *Quarterly Journal of Economics*, 122, 39-72.
- [16] Hansen, G.D. (1993): “The Cyclical and Secular Behavior of the Labour Input: Comparing Efficiency Units and Hours Worked,” *Journal of Applied Econometrics*, 8, 71-80.
- [17] Hogan, C., J. Lunney, J. Gabel, and J. Lynn (2001): “Medicare Beneficiaries Costs of Care in the Last Year of Life,” *Health Affairs*, 20, 188-195.

- [18] Hubbard, G.J., J.S. Skinner and S.P. Zeldes (1994): “The Importance of Precautionary Motives for Explaining Individual and Aggregate Savings,” in Carnegie-Rochester Conference Series on Public Policy, Vol 40, ed. by A.H. Meltzer and C.I. Plosser. Amsterdam: North-Holland, pp. 59-125.
- [19] Imrohoroglu, A., S. Imrohoroglu and D. Joines (1995): “A Life-Cycle Analysis of Social Security,” *Economic Theory*, 6, 83-114.
- [20] Jeske, K. and S. Kitao (2009): “U.S. Tax Policy and Health Insurance Demand: Can a Regressive Policy Improve Welfare?” *Journal of Monetary Economics*, 56, 210-221.
- [21] Kashihara, D. and K. Carper (2008): “National Health Care Expenses in the US Civilian Non-institutionalized Population, 2009” Medical Expenditure Panel Survey Statistical Brief #229.
- [22] Lillard, E.L. and R.W. Willis (1978): “Dynamic Aspects of Earnings Mobility,” *Econometrica*, 46, 985-1012.
- [23] Meghir, C. and L. Pistaferri (2004), “Income Variance Dynamics and Heterogeneity,” *Econometrica*, 72, 1-32.
- [24] Muurinen, J. (1982): “Demand for Health: A Generalized Grossman Model,” *Journal of Health Economics*, 1, 5-28.

- [25] Palumbo, M.G. (1999): “Uncertain Medical Expenses and Precautionary Saving Near the End of the Life-Cycle,” *Review of Economic Studies*, 66, 395-421.
- [26] Picone, G., M. Uribe and M. Wilson (1998): “The Effect of Uncertainty on the Demand for Medical Care, Health Capital and Wealth,” *Journal of Health Economics*, 17, 171-185.
- [27] Reinhardt, U.E., P.S. Hussey and G.F. Anderson (2004): “U.S. Health Care Spending in an International Context,” *Health Affairs*, 23, 10-25.
- [28] Rust, J. and C. Phelan (1997): “How Social Security and Medicare Affect Retirement Behavior in a World of Incomplete Markets,” *Econometrica*, 65, 781-831.
- [29] Smith, J. (2003): “Health and SES over the Life-Course,” unpublished mimeo.
- [30] Suen, R. (2006): “Technological Advance and the Growth in Health Care Spending,” unpublished mimeo.
- [31] Wagstaff, A. (1986): “The Demand for Health: Some New Empirical Evidence,” *Journal of Health Economics*, 1, 5-28.
- [32] Wagstaff, A. (1993): “The Demand for Health: An Empirical Reformulation of the Grossman Model,” *Health Economics*, 2, 189-198.



[33] Yogo, M. (2008): “Portfolio Choice in Retirement: Health Risk and the Demand for Annuities, Housing and Risky Assets,” unpublished mimeo.

## 9 Appendix 1: Derivation of Equation (10)

We derive the FOCs for the individual’s problem (9) as follows:

$$c_j : \beta^{j-1} \left[ \prod_{k=1}^j \varphi_k \right] \frac{\partial U}{\partial c_j} = \phi_j \quad (12)$$

$$m_j : \phi_j = \mu_j MPM_j \quad (13)$$

$$h_{j+1} : \beta^j \left[ \prod_{k=1}^{j+1} \varphi_k \right] \frac{\partial U}{\partial h_{j+1}} - \mu_j + (1 - \delta_{h_j})\mu_{j+1} - \phi_{j+1} w \varepsilon_{j+1} \frac{\partial s_{j+1}}{\partial h_{j+1}} = 0 \quad (14)$$

where  $\phi_j$  and  $\mu_j$  are the associated Langrangian multipliers for the budget constraint equation (5) and the skill accumulation equation (7), respectively. We also have

$$\begin{aligned} \frac{\partial U}{\partial c_j} &= \frac{\rho F}{c_j} \\ \frac{\partial U}{\partial h_j} &= (1 - \lambda)[\lambda(c_j^\rho l_j^{1-\rho})^\psi + (1 - \lambda)h_j^\psi]^{\frac{1-\sigma}{\psi}-1} h_j^{\psi-1} \\ MPM_j &= \theta \xi B m_j^{\theta\xi-1} v_j^{(1-\theta)\xi} \\ \text{with } F &= \lambda[\lambda(c_j^\rho l_j^{1-\rho})^\psi + (1 - \lambda)h_j^\psi]^{\frac{1-\sigma}{\psi}-1} (c_j^\rho l_j^{1-\rho})^\psi. \end{aligned}$$

Substituting (12) and (13) into (14), we obtain equation (10).